Math 171 Test 1

Place your name in the upper right hand corner. Place your answers in the blank space on the left when appropriate. Show your work on a separate sheet. You may not use your calculator to compute limits or derivatives. Have fun ☺

Graph the following function. Indicate all asymptotes, corners and intercepts. You may assume that *a* is greater than *b*.

1) $f(x) = \dfrac{x^2 - b^2}{x^2 - a^2}$

Derive the following identities. Hint: look at the formulae first.

2) $\sin(2t) = 2\sin(t)\cos(t)$ 3) $\cos(2t) = \cos^2(t) - \sin^2(t)$

4) Consider the following function, $T(t) = A + A\cos(\omega t + \pi)$. A and ω are positive constants.

_____ a) What is the maximum value of *T*?
_____ b) Give one value of *t* for which this will occur.
_____ c) What is the minimum value of *T*?
_____ d) Give one value of *t* for which this will occur.

For questions (5) through (8) compute the limit. If the limit does not exist indicate positive or negative infinity if possible. If the last option is not possible tell me why the limit does not exist. You must do this by hand, no calculator.

_____ 5) $\lim\limits_{x \to -2} \dfrac{x^2 - 4}{x + 2}$

_____ 6) $\lim\limits_{x \to 0} \dfrac{\sqrt{(b^2 - x)} - b}{\sqrt{(a^2 - x)} - a}$

_____ 7) $\lim\limits_{x \to \infty} \left(\sqrt{x^2 + ax} - \sqrt{x^2 + bx} \right)$

_____ 8) $\lim\limits_{x \to 1} \dfrac{x^2 + x - 2}{x^2 - 3x + 2}$

9) Consider a star of radius *r*. Find a function that represents the volume of stellar material required to increase the size of the star from a radius of *r* to a radius of $r + a$.

_____ 10) Solve the following equation $e^x + e^{-x} = \pi$.

11) Find the equation of the tangent line to the curve $y = \dfrac{x^3}{a} + x^2$ at the point $x = a$.

Formulae

$$\frac{d}{dx} f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad \frac{d}{dx} x^n = nx^{n-1}$$

$$e^{ix} = \cos(x) + i\sin(x) \qquad (e^{ix})^n = \cos(nx) + i\sin(nx)$$