Part A Solutions

Problem #1

\[ 5 + x = 12 \]  
First subtract 5 from both sides of the equation.

\[ 5 + x - 5 = 12 - 5 \]  
Now combine like terms.

\[ x = 7 \]

Problem #2

\[ 6x = 36 \]  
First divide both sides of the equation by 6.

\[ \frac{6x}{6} = \frac{36}{6} \]  
Next, simplify.

\[ x = 6 \]

Problem #3

\[ 3 = \frac{z}{-5} \]  
First multiply both sides of the equation by \(-5\).

\[ 3(-5) = \frac{z}{-5}(-5) \]  
Next simplify.

\[ -15 = \frac{z}{-5}(-5) \]

\[ -15 = z \]
Problem #4

\[ \frac{(3+5)^2 + |\!-2|}{2(5-2^3)} \]
Begin simplifying the numerator by doing the addition within the parentheses.

\[ \frac{(8)^2 + |-2|}{2(5-2^3)} \]
Continue by applying the exponent.

\[ \frac{64 + |\!-2|}{2(5-2^3)} \]
Next evaluate the absolute value.

\[ \frac{64 + 2}{2(5-2^3)} \]
Now do the addition in the numerator.

\[ \frac{66}{2(5-2^3)} \]
Next simplify the denominator, beginning within the parentheses.

\[ \frac{66}{2(5-8)} \]
Now do the subtraction in the denominator.

\[ \frac{66}{2(-3)} \]
Do the multiplication in the denominator.

\[ \frac{66}{-6} \]
Now do the division.

\[ -11 \]
Problem #5

Since $x = 4$, $y = 3$, and $z = 5$:

$(-z)^2 + 4x - 7y$  First substitute the given values for $x$, $y$, and $z$.

$(-5)^2 + 4(4) - 7(3)$  Now apply the order of operations.

$25 + 4(4) - 7(3)$  No parentheses so calculate the exponent first.

$25 + 16 - 7(3)$  Next do multiplication and division from left to right.

$25 + 16 - 21$

$41 - 21$  Now do addition and subtraction from left to right.

$20$
Problem #6

\[ 3(x - 1) + 7x = -5 \]  First expand the term to the left (multiply \( x - 1 \) by 3).

\[ 3x - 3 + 7x = -5 \]  Now combine like terms on the left side of the equation (\( 3x + 7x = 10x \)).

\[ 10x - 3 = -5 \]  Next isolate the variable term by adding 3 to both sides of the equation.

\[ 10x - 3 + 3 = -5 + 3 \]

\[ 10x = -2 \]  Now isolate the variable \( x \) by multiplying both sides of the equation by the reciprocal of the coefficient of \( x \).

\[ 10x \left( \frac{1}{10} \right) = -2 \left( \frac{1}{10} \right) \]

\[ \frac{10}{10} x = -\frac{2}{10} \]

\[ x = -\frac{1}{5} \]
Problem #7

\[ A = P(1 + rt) \] We must isolate \( r \) on one side of the equation. Begin by dividing both sides of the equation by \( P \).

\[ \frac{A}{P} = \frac{R(1 + rt)}{R} \]

\[ \frac{A}{P} = 1 + rt \] Now subtract 1 from each side of the equation.

\[ \frac{A}{P} - 1 = \lambda + rt - \lambda \]

\[ \frac{A}{P} - 1 = rt \] Next multiply both sides of the equation by \( \frac{1}{t} \) (or divide by \( t \)).

\[ \left( \frac{A}{P} - 1 \right) \frac{1}{t} = \left( \lambda \frac{1}{\lambda} \right) \frac{1}{t} \]

\[ \left( \frac{A}{P} - 1 \right) \frac{1}{t} = r \]
Problem #8

\[ x + \frac{x}{7} = \frac{x}{3} + \frac{4}{3} \]

Eliminate the fractions by multiplying both sides of the equation by 21, which is the least common multiple of 3 and 7.

\[ 21 \left( x + \frac{x}{7} \right) = 21 \left( \frac{x}{3} + \frac{4}{3} \right) \]

\[ 21x + \frac{21x}{7} = \frac{21x}{3} + \frac{21(4)}{3} \]

\[ 21x + \frac{21}{7}x = \frac{21}{3}x + \frac{21}{3}(4) \]

\[ 21x + 3x = 7x + 7(4) \]

\[ 24x = 7x + 28 \]

Now subtract 7x from both sides of the equation.

\[ 24x - 7x = 7x + 28 - 7x \]

\[ 17x = 28 \]

Next multiply both sides of the equation by \( \frac{1}{17} \) (or divide by 17).

\[ \frac{17x}{17} = \frac{28}{17} \]

\[ x = \frac{28}{17} \]
Problem #9

Ernie has saved $1,202.50 so far. He is saving to buy a new stereo system and has figured that he has saved 65% of the selling price of the system. What is the selling price of the stereo system Ernie wants to buy?

Let $P =$ the selling price of the stereo system. Since Ernie has saved 65% of the selling price and he has saved $1,202.50, 65% of $P$ is 1202.5. This translates to the equation:

\[ 65 \left( \frac{1}{100} \right) P = 1202.5 \quad \text{We want to solve for } P. \]

\[ \frac{65}{100} P = 1202.5 \quad \text{Multiply both sides of the equation by the reciprocal of the coefficient of } P. \]

\[ \left( \frac{100}{65} \right) \frac{65}{100} P = \left( \frac{100}{65} \right) 1202.5 \]

\[ P = \frac{120250}{65} = 1850 \quad \text{So the selling price is } \$1,850.00 \]

Problem #10

A recent rental bill for a car rental was $235, including a $100 rental fee and the rest for mileage at $0.25 per mile. How many miles did the renter drive the car?

If we let $m$ be the number of miles the renter drove, then $100 + 0.25m$ represents the total cost to rent the car which is $235. So we must solve the equation $100 + 0.25m = 235$.

\[ 100 + 0.25m = 235 \quad \text{First subtract 100 from each side of the equation.} \]
\[ 100 + 0.25m - 100 = 235 - 100 \]
\[ 0.25m = 135 \quad \text{Next divide both sides of the equation by 0.25} \]

\[ \frac{0.25m}{0.25} = \frac{135}{0.25} \]

\[ m = 540, \quad \text{So the driver drove 540 miles.} \]
Problem #11

To solve the system by graphing, graph each equation and determine the points of intersection. These points make up the solution set.

\[
\begin{align*}
-x + 3y &= -11 \\
3x - y &= 17
\end{align*}
\]

Since each equation represents a straight line, graph the first equation by finding two points that satisfy the equation. When \(x = -1\),

\[-x + 3y = -11 \text{ becomes } -(\-1\)+ 3y = -11 \text{ and solving for } y \text{ we get;}

\[1+3y-1=-11-1 \]

\[3y = -12 \]

\[\frac{3y}{3} = -\frac{12}{3} \]

\[y = -4 \text{ so the point } (-1, -4) \text{ is a point on the first line.} \]

When \(x = 2\),

\[-x + 3y = -11 \text{ becomes } -(\2)+ 3y = -11 \text{ and solving for } y \text{ we get;}

\[-2+3y+2=-11+2 \]

\[3y = -9 \]

\[\frac{3y}{3} = -\frac{9}{3} \]

\[y = -3 \text{ so the point } (2, -3) \text{ is also a point on the first line.} \]

We can now plot these two points to graph the first line.
Now we graph the second equation. When $x = 3$,
$3x - y = 17$ becomes $3(3) - y = 17$ and solving for $y$ we get;
$9 - y - 9 = 17 - 9$
$-y = 8$
$\Rightarrow y = \frac{8}{-1}$
$y = -8$ so the point $(3, -8)$ is a point on the second line.

When $x = 7$,
$3x - y = 17$ becomes $3(7) - y = 17$ and solving for $y$ we get;
$21 - y - 21 = 17 - 21$
$-y = -4$
$\Rightarrow y = \frac{-4}{-1}$
$y = 4$ so the point $(7, 4)$ is also a point on the second line.

We can now plot these two points to graph the second line.

We can now estimate the point of intersection. It appears to be about $(5, -2)$. If we check this proposed solution in each of the original equations we will confirm that it satisfies both.
Problem #12

\[
\begin{align*}
7x - 11 &= 3y \\
6x &= 2y + 34
\end{align*}
\]

To solve by substitution, we begin by solving one equation for one of the variables. Let's pick the first equation \(x - 11 = 3y\) and solve for \(x\).

\[
x = 3y + 11
\]

Now substitute this expression for \(x\) into the other equation \(6x = 2y + 34\) and solve for \(y\).

\[
6(3y + 11) = 2y + 34
\]

\[
18y + 66 = 2y + 34
\]

\[
18y + 66 - 66 = 2y + 34 - 66
\]

\[
18y = 2y - 32
\]

\[
18y - 2y = 2y - 32 - 2y
\]

\[
16y = -32
\]

\[
\frac{16y}{16} = \frac{-32}{16}
\]

\[
y = -2
\]

Now we can substitute this value for \(y\) in either equation and solve for \(x\).

\[
6x = 2(-2) + 34
\]

\[
6x = -4 + 34
\]

\[
6x = 30
\]

\[
\frac{6x}{6} = \frac{30}{6}
\]

\[
x = 5
\]

So the proposed solution is the point \((5, -2)\). This proposed solution satisfies all original equations so the solution set is \{\((5, -2)\)\}.
Problem #13

\[
\begin{aligned}
\frac{1}{3}x + y &= 7 \\
\frac{2}{3}x - y &= -4
\end{aligned}
\]

To solve the system by addition (elimination), add the left side of the first equation to the left side of the second equation and set that sum equal to the sum of the right sides of each equation. This will eliminate the \(y\) variable since they have opposite coefficients.

\[
\left(\frac{1}{3}x + y\right) + \left(\frac{2}{3}x - y\right) = (7) + (-4)
\]

\[
\frac{1}{3}x + \frac{2}{3}x = 7 - 4
\]

\[
\frac{1}{3}x + \frac{2}{3}x = 3 \quad \text{Now that the } y \text{ variable is eliminated, we solve for the } x \text{ variable.}
\]

\[
\frac{3}{3}x = 3
\]

\[
x = 3 \quad \text{Now substitute this value for } x \text{ in either original equation and solve for } y.
\]

\[
\frac{1}{3}x + y = 7
\]

\[
\frac{1}{3}(3) + y = 7
\]

\[
1 + y = 7
\]

\[
y = 6 \quad \text{So the proposed solution is the point (3, 6). This proposed solution satisfies each original equation so the solution set is \{(3, 6)\}.}
\]
Problem #14

Ice cream cones sell for $1.10 and sundaes sell for $3.25 at one store. If the receipts for a total of 115 cones and sundaes were $216.80, how many of each were sold?

Let \( c \) be the number of cones sold and let \( s \) be the number of sundaes sold. The number of cones and sundaes sold is 115 so \( c + s = 115 \). The receipts for these 115 treats was $216.80 so \( 1.10c + 3.25s = 216.80 \) and we have the following system of equations:

\[
\begin{align*}
  c + s &= 115 \\
  1.10c + 3.25s &= 216.80
\end{align*}
\]

We need one of the variables to have opposite coefficients so we can eliminate it. We can get this by multiplying both sides of the first equation by \(-1.1\) to get:

\[
\begin{align*}
  -1.1c - 1.1s &= 115(-1.1) \\
  1.10c + 3.25s &= 216.80
\end{align*}
\]

\[
\begin{align*}
  -1.1c - 1.1s &= -126.5 \\
  1.1c + 3.25s &= 216.8
\end{align*}
\]

\[-1.1c - 1.1s + 1.1c + 3.25s = -126.5 + 216.8 \]

\[-1.1s + 3.25s = 90.3 \]

\[
2.15s = 90.3
\]

\[
\frac{2.15s}{2.15} = \frac{90.3}{2.15}
\]

\( s = 42 \) So 42 sundaes were sold. Now substitute this value for \( s \) into either original equation and solve for \( c \).

\[
c + s = 115
\]

\[
c + (42) = 115
\]

\[
c + 42, 42 = 115 - 42
\]

\( c = 73 \) So 73 cones were sold. The proposed solution, \( c = 73 \) and \( s = 42 \) satisfies both equations and checks in the wording of the problem so it is the solution.
Problem #15

\[(2x^2 - 3x + 1) + (x^2 + 2x - 5)\]
Write the first polynomial without parentheses.

\[2x^2 - 3x + 1 + x^2 + 2x - 5\]
Next add each term of the second polynomial to the first.

\[2x^2 - 3x + 1 + x^2 + 2x + (-5)\]

\[2x^2 - 3x + 1 + x^2 + 2x - 5\]
Now combine the like terms.

\[(2x^2 + x^2) + (-3x + 2x) + (1 - 5)\]

\[3x^2 + (-x) + (-4)\]

\[3x^2 - x - 4\]

Problem #16

\[(t^3 + 2t - 4) - (2t^2 - t + 2)\]
Write the first polynomial without parentheses.

\[t^3 + 2t - 4 - 2t^2 + t - 2\]
Next subtract each term of the second polynomial from the first.

\[t^3 + 2t - 4 - 2t^2 - (-t) - 2\]

\[t^3 + 2t - 4 - 2t^2 + t - 2\]
Now combine like terms.

\[t^3 - 2t^2 + 3t - 6\]

Problem #17

\[(5x)(7x^3)\]
To multiply monomials, we multiply the numeric coefficients first.

\[(5)(x)(7)(x^3)\]

\[(5)(7)(x)(x^3)\]

\[35(x)(x^3)\]
Then we multiply the variable parts by adding the exponents.

\[35(x^1)(x^3)\]

\[35x^{1+3}\]

\[35x^4\]
Problem #18

\[(7x)(3x^2 + x + 5)\]  To multiply polynomials, we multiply each term of the second polynomial by each term of the first polynomial.

\[(7x)(3x^2) + (7x)(x) + (7x)(5)\]
\[21x^3 + 7x^2 + 35x\]

Problem #19

\[(2x - 4)(4x^2 - 3x + 2)\]  Multiply each term of the second polynomial by each term of the first.

\[(2x)(4x^2) + (2x)(-3x) + (2x)(2) + (-4)(4x^2) + (-4)(-3x) + (-4)(2)\]
\[8x^3 + (-6x^2) + 4x + (-16x^2) + 12x + (-8)\]  Now combine like terms.
\[8x^3 + (-6x^2 - 16x^2) + (4x + 12x) - 8\]
\[8x^3 - 22x^2 + 16x - 8\]

Problem #20

\[(x + 4)(2x - 7)\]  Multiply each term of the second polynomial by each term of the first.

\[(x)(2x) + (x)(-7) + (4)(2x) + (4)(-7)\]
\[2x^2 + (-7x) + 8x + (-28)\]  Now combine like terms.
\[2x^2 + (-7x + 8x) - 28\]
\[2x^2 + x - 28\]

Problem #21

\[24a^2b^3 + 8ab^2\]  Note that 8 is the greatest common factor of the numeric coefficients 24 and 8, \(a\) is the greatest common factor of \(a^2\) and \(a\), and \(b^2\) is the greatest common factor of \(b^3\) and \(b^2\) so \(8ab^2\) is the greatest common factor of \(24a^2b^3\) and \(8ab^2\).

\[\frac{(8ab^2)(3ab)}{24a^2b^3} + \frac{(8ab^2)(1)}{8ab^2}\]
\[\frac{(8ab^2)(3ab + 1)}{}\]
Problem #22

\[ 6y^2 + 2y + 9y + 3 \]
\[ (6y^2 + 2y) + (9y + 3) \]
\[ (2y)(3y + 1) + (3)(3y + 1) \]
\[ (3y + 1)(2y + 3) \]

First group terms with common factors.
Next factor out each groups' greatest common factor.
Now factor out the common factor \((3y + 1)\) of the addends.

Problem #23

\[ x^2 + 6x - 7 \]

This is a trinomial of the form \(ax^2 + bx + c\) where \(a = 1\) so we must find two numbers \(m\) and \(n\) such that \(m + n = b\) (6 in this problem) and \(mn = c\) (−7 here). The factorization will then be \((x + m)(x + n)\).

Since the product of the two numbers \(m\) and \(n\) is negative \((mn = -7)\), one of these numbers must be negative and the other must be positive. Since the sum \(m + n = 6\) which is positive, the number with the greatest absolute value must be positive. Note that \((7)(-1) = -7\) and \((7) + (-1) = 6\) so we let \(m = 7\) and \(n = -1\), giving the factorization \((x + 7)(x - 1)\).

Problem #24

\[ 4x^2 + 8x + 3 \]

This is a trinomial of the form \(ax^2 + bx + c\) where \(a \neq 1\) so we must find two numbers \(m\) and \(n\) such that \(m + n = b\) (8 in this problem) and \(mn = ac\) (12 here). Then we must use this \(m\) and \(n\) to split \(8x\) into two terms and factor by grouping.

Since the product of the two numbers \(m\) and \(n\) is positive \((mn = 12)\), both of these numbers must be positive or they must both be negative. Since the sum \(m + n = 8\) which is positive, both numbers must be positive. Note that \((2)(6) = 12\) and \((2) + (6) = 8\) so we let \(m = 2\) and \(n = 6\), and split \(8x\) into \(2x + 6x\). This allows us to factor by grouping.

\[ 4x^2 + 8x + 3 \]
\[ 4x^2 + 2x + 6x + 3 \]
\[ (4x^2 + 2x) + (6x + 3) \]
\[ 2x(2x + 1) + 3(2x + 1) \]
\[ (2x + 1)(2x + 3) \]
Problem #25

\[12x^3 + 38x^2 + 6x \quad \text{First factor out the greatest common factor } 2x.\]
\[2x(6x^2 + 19x + 3) \quad \text{Next factor } [6x^2 + 19x + 3] \text{ by grouping.}\]
\[2x\left[(6x^2 + x) + (18x + 3)\right]\]
\[2x\left[x(6x + 1) + 3(6x + 1)\right]\]
\[2x((6x + 1)(x + 3))\]
\[2x(6x + 1)(x + 3)\]

Problem #26

\[a^2 - 36 \quad \text{This is the difference of two squares } a^2 - 6^2 \text{ so it can be factored directly}\]
\[\text{from the formula } A^2 - B^2 = (A + B)(A - B). \text{ Here } A = a \text{ and } B = 6 \text{ so the}\]
\[\text{factorization is } (a + 6)(a - 6).\]

\[\frac{a^2 - 36}{a^2 - 6^2}\]

\[\frac{a^2 - 6^2}{A^2 - B^2} = \frac{(a + 6)(a - 6)}{(A + B)(A - B)}\]

Problem #27

\[x^3 - 125 \quad \text{This is the difference of two cubes } x^3 - 5^3 \text{ so it can be factored directly}\]
\[\text{from the formula } A^3 - B^3 = (A - B)(A^2 + AB + B^3). \text{ Here } A = x \text{ and } B = 5 \text{ so the}\]
\[\text{factorization is } (x - 5)(x^2 + 5x + 25).\]

\[\frac{x^3 - 125}{x^3 - 5^3}\]

\[\frac{x^3 - 5^3}{A^3 - B^3} = \frac{(x - 5)(x^2 + 5x + 25)}{(A^2 + AB + B^3)}\]
Problem #28

\[ 2x^2 - x - 6 = 0 \]

First we factor the trinomial on the left side of the equation.

\[ 2x^2 - 4x + 3x - 6 = 0 \]

\[ (2x^2 - 4x) + (3x - 6) = 0 \]

\[ 2x(x - 2) + 3(x - 2) = 0 \]

\[ (x - 2)(2x + 3) = 0 \]

Now we apply the zero product principle.

\[ (x - 2) = 0 \] or \[ (2x + 3) = 0 \]

\[ x - 2 = 0 \] or \[ 2x + 3 = 0 \]

\[ x - 2 + 2 = 0 + 2 \] or \[ 2x + 3 - 3 = 0 - 3 \]

\[ x = 2 \] or \[ 2x = -3 \]

\[ x = 2 \] or \[ \frac{2x}{2} = \frac{-3}{2} \]

\[ x = 2 \] or \[ x = -\frac{3}{2} \]

So the solution set is \( \{2, -\frac{3}{2}\} \)

Problem #29

\[ x^2 + 5x + 6 = 0 \]

Here \( a = 1 \), \( b = 5 \), and \( c = 6 \) so applying the quadratic formula we get:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ x = \frac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2(1)} = \frac{-5 \pm \sqrt{25 - 24}}{2} = \frac{-5 \pm \sqrt{1}}{2} = \frac{-5 \pm 1}{2} \]

\[ x = \frac{-5 + 1}{2} = \frac{-4}{2} = -2 \] or \[ x = \frac{-5 - 1}{2} = \frac{-6}{2} = -3 \]

So the solution set is \( \{-2, -3\} \)
Problem #30

\[ y = x^2 + 2x - 3 \]
Here \( a = 1, \ b = 2, \) and \( c = -3 \) and the vertex is the point \( \left( \frac{-b}{2a}, f\left( \frac{-b}{2a} \right) \right) \).

\[
\frac{-b}{2a} = \frac{-2}{2(1)} = -1 \quad \text{and} \quad f(-1) = (-1)^2 + 2(-1) - 3 = 1 - 2 - 3 = -4, \quad \text{so the vertex is} \ ( -1, \ -4 ).
\]

The \( y \)-intercept occurs at the point where \( x = 0; \)
\[ y = 0^2 + 2(0) - 3 = -3 \quad \text{so the \( y \)-intercept is} \ -3 \quad \text{and it occurs at the point} \ (0, \ -3).\]

The \( x \)-intercepts occur at the points where \( y = 0; \)
\[ 0 = x^2 + 2x - 3 = (x + 3)(x - 1) \]
\[ (x + 3) = 0 \quad \text{or} \quad (x - 1) = 0 \]
\[ x = -3 \quad \text{or} \quad x = 1 \]
So the \( x \)-intercepts occur at the points \((-3, \ 0)\) and \((1, \ 0)\).

To find one other point on the graph we pick a value for \( x \) and calculate \( y; \)
Let \( x = 2, \) then \[ y = 2^2 + 2(2) - 3 = 4 + 4 - 3 = 5, \] so the point \((2, \ 5)\) is on the graph.

Now we use this information to plot the graph.
Problem #31

\[7 - 5x \leq 42\]  
We solve the inequality as if it was an equation with the exception that if we multiply or divide both sides by a negative number, we reverse the inequality symbol.

\[7 - 5x - 7 \leq 42 - 7\]
\[-5x \leq 35\]
\[x \geq \frac{35}{-5}\]
\[x \geq -7\]  
Note that the symbol was reversed here (\(\leq\) to \(\geq\)) since we divided by \(-5\).

In set builder notation we write the solution set as \(\{x | x \geq -7\}\).

Problem #32

\[8x + 6(2x - 7) \geq 2x - 6\]
\[8x + 12x - 42 \geq 2x - 6\]
\[20x - 42 \geq 2x - 6\]
\[20x - 42 + 42 \geq 2x - 6 + 42\]
\[20x \geq 2x + 36\]
\[20x - 2x \geq 2x + 36 - 2x\]
\[18x \geq 36\]
\[x \geq \frac{36}{18}\]
\[x \geq 2\]  
In set builder notation we write \(\{x | x \geq 2\}\).

Problem #33

The line \(y = \frac{1}{2}x - 8\) has slope \(m = \frac{1}{2}\). We see this since the equation is given in slope-intercept form, \(y = mx + b\), where \(m\) represents the slope of the line and \(b\) represents the \(y\)-intercept. Since parallel lines have the same slope, the slope of the line we are looking for is also \(\frac{1}{2}\). We use this slope and the point \((1, -6)\) in the point slope form for the equation of a line, \((y - y_i) = m(x - x_i)\), with \(m = \frac{1}{2}\), \(y_i = -6\), and \(x_i = 1\) to get \((y - (-6)) = \frac{1}{2}(x - 1)\) or \((y + 6) = \frac{1}{2}(x - 1)\).
Problem #34

We use the given slope $m = 3$ and the point $(3, 2)$ in the point slope form for the equation of a line, $(y - y_1) = m(x - x_1)$, with $m = 3$, $y_1 = 2$, and $x_1 = 3$ to get $(y - 2) = 3(x - 3)$.

Problem #35

The line $y = \frac{9}{17} x + 27$ has slope $m = \frac{9}{17}$. Since perpendicular lines have negative reciprocal slopes, a line perpendicular to $y = \frac{9}{17} x + 27$ has slope $m = -\frac{17}{9}$.