Part B Problems

1. Solve the following system of equations:
   \[
   \begin{align*}
   2x + y - 2z &= -1 \\
   3x - 3y - z &= 5 \\
   x - 2y + 3z &= 6 
   \end{align*}
   \]

2. Solve the following system of equations using matrices and row operations:
   \[
   \begin{align*}
   3x - 4y &= 4 \\
   2x + 2y &= 12 
   \end{align*}
   \]

3. Solve the following system of equations using Cramer’s Rule:
   \[
   \begin{align*}
   7x - 11y &= 3 \\
   9x + 2y &= 20 
   \end{align*}
   \]

4. Evaluate the following determinant:
   \[
   \begin{vmatrix}
   3 & 5 & 1 \\
   6 & -2 & 2 \\
   8 & -1 & 4 
   \end{vmatrix}
   \]

5. Given \( f(x) = 2x^2 + x - 5 \), \( g(x) = x - 4 \) find:
   a. \((f + g)(x)\)
   b. \((f - g)(x)\)
   c. \((fg)(x)\)
   d. \((f/g)(x)\)

6. Solve the following compound inequality giving the answer in interval notation:
   \[
   2x - 5 \leq -11 \quad \text{or} \quad 5x + 1 \geq 6
   \]
7. Given $f(x) = 2x^3 - 5$, $g(x) = x - 4$ find:
   a. $(f \circ g)(x)$
   b. $(f \circ g)(3)$
   c. $(g \circ f)(x)$
   d. $f^{-1}(x)$
   e. $g^{-1}(x)$

8. Solve the following inequality giving the answer in interval notation:
   
   $$-3(a + 2) > 2(a + 1)$$

9. Solve the following compound inequality giving the answer in interval notation:
   
   $$2x + 1 > 4x - 3 \quad \text{and} \quad x - 1 \geq 3x + 5$$

10. Solve the equation: $|3x + 2| = 16$

11. Solve the following inequality giving the answer in interval notation:
   
   $$|5x - 2| > 13$$

12. Graph the following inequality: $x - 3y \leq 6$

13. Graph the solution set of the following system of inequalities:

   $$4x - 5y \geq -20$$
   $$x \geq -3$$
14. Use linear programming to solve the following problem.

You are taking a test that contains computation problems worth 6 points each and word problems worth 10 points each. You can do a computation problem in 2 minutes and a word problem in 4 minutes. You have 40 minutes to take the test and may answer no more than 12 problems. Assuming that you answer every attempted problem correctly, how many of each type of problem must you do to maximize your score and what is the maximum score? (Write the objective function and constraints, graph the constraints labeling all vertices, and solve the problem.)

15. Find the domain of the following function:

\[ f(x) = \sqrt{8 - 2x} \]

16. Simplify the following:

a. \( \sqrt[4]{-32(x - 2)^5} \)

b. \( \sqrt[4]{(x + 5)^4} \)

17. Use rational exponents to simplify the expression and write the result in radical notation. Assume that all variables represent positive real numbers:

\( \sqrt[3]{2x} \)

18. Simplify, writing the answer with no negative exponents. Assume that all variables represent positive real numbers:

\( \left( \frac{1}{2y^5} \right)^4 \div \sqrt[3]{y^{10}} \)

19. Multiply and simplify. Assume that all variables represent positive real numbers:

\( \sqrt[3]{x - 6} \cdot \sqrt[3]{(x - 6)^2} \)
20. Perform the indicated operations and simplify:
   a. $6\sqrt{7} - 3\sqrt{x} + 2\sqrt{7} + 5\sqrt[4]{x}$
   b. $4\left(\sqrt[4]{x^4 \cdot y^2}\right) + 5x\sqrt[4]{xy^2}$
   c. $\frac{13y^7}{x^{12}}$
   d. $\frac{\sqrt{50xy}}{2\sqrt{2}}$

21. Rationalize each denominator:
   a. $\frac{5}{4\sqrt{x}}$  
   b. $\frac{3\sqrt{x} + \sqrt{y}}{\sqrt{y} - 3\sqrt{x}}$

22. Find the solution set to the following equation:
   $\sqrt[4]{x^4 + 4x^2 - 4} = -x$

23. Find the solution set to the following equation:
   $\sqrt{y + 7} + 3 = \sqrt{y + 4}$

24. Divide. Give the answer in $a + bi$ form.
   $\frac{4 - 3i}{7 + 2i}$

25. Complete the square to find the solution set:
   $8x^2 - 10x = 3$

26. For the points $(-3, 5)$ and $(-5, -5)$
   a. Find the exact distance between the points.
   b. Find the midpoint of the line segment joining the points.
27. Use the quadratic formula to find both solutions in the solution set of the following equation. Give complex solutions in $a + bi$ form.

$$-7x = x^2 - 4$$

28. Use the quadratic formula to find both solutions in the solution set of the following equation. Give complex solutions in $a + bi$ form.

$$2x^2 - 5x + 4 = 0$$

29. Given the following quadratic function:

$$f(x) = (x - 1)^2 - 4$$

a. Find the coordinates of the vertex.
b. Find all $x$ and $y$ intercepts.
c. Does the function have a maximum or minimum value?
d. What are the coordinates of the functions minimum or maximum point?

30. Find the solution set for the following equation:

$$x^4 - 68x^2 + 256 = 0$$

31. Solve the following inequality:

$$567 > 7x^2$$

32. Solve the following inequality:

$$\frac{2}{x + 5} \geq \frac{1}{x}$$

33. a. Convert $56^\circ$ to radians. b. Convert $\frac{7\pi}{12}$ radians to degrees.

34. Given a right triangle with hypotenuse measuring 13 units and legs measuring 5 and 12 units. If $\theta$ is the angle formed by the hypotenuse and the 12 unit leg, find:

$$\sin(\theta), \cos(\theta), \tan(\theta), \sec(\theta), \csc(\theta), \cot(\theta)$$

35. In triangle $ABC$, angle $A$ measures 60 degrees and angle $B$ measures 35 degrees. Find the measure of angle $C$. 
36. In triangle $ABC$, angle $A$ measures 44 degrees, angle $B$ measures 30 degrees, and side $a$ measures 7 units. Use the law of sines to find the measure of side $b$ to the nearest thousandth of a unit. Note: side $a$ is located opposite angle $A$, side $b$ is located opposite angle $B$, and side $c$ is located opposite angle $C$.

37. In triangle $ABC$, angle $A$ measures 75 degrees, side $b$ measures 6 units, and side $c$ measures 12 units. Use the law of cosines to find the measure of side $a$ to the nearest hundredth of a unit. Note: side $a$ is located opposite angle $A$, side $b$ is located opposite angle $B$, and side $c$ is located opposite angle $C$.

38. Rewrite the following expressions in logarithmic form (if given in exponential form) or in exponential form (if given in logarithmic form):
   
   a. $\log_{8} y = 10$
   b. $9^{x} = 42$

39. Graph the following function, labeling at least three points:

$$f(x) = 7^{x}$$

40. What is the balance in your account after six years if:
   
   a. You invest $4,000.00 at 7% compounded monthly?
   b. You invest $4,000.00 at 7% compounded continuously?

41. Write the following as a single logarithm:

$$2 \log_{a} x - 3 \log_{b} x + 5 \log_{c} x$$

42. Graph the following function, labeling at least three points:

$$f(x) = \log_{7} x$$

43. Solve for $x$ rounding to four decimal places if necessary:

$$4^{x+3} = 1024$$

44. Solve for $x$ rounding to four decimal places if necessary:

$$8^{6x^2 - 7x} = 32768$$

45. Solve for $x$ rounding to four decimal places if necessary:

$$\ln 23x = \ln 69$$
46. Solve for $x$ rounding to four decimal places if necessary:

$$\log_9 (x + 5) - \log_9 (2x + 9) = 0$$

47. Solve for $a$ rounding to four decimal places if necessary:

$$4 \log_3 a + 5 \log_3 a = 6 \log_3 a + 3$$

48. For each equation below:

   a. Identify the conic section.
   b. Write the equation in standard form.
   c. Give the coordinates of all vertices and centers.
   d. Graph the conic section labeling all vertices, centers, and two additional points.

   $$4x^2 + 9y^2 - 16x - 18y = 11$$
   $$y^2 - 4x^2 - 4y - 8x = 4$$

49. Solve the following system of equations:

   $$3x^2 + 2y^2 = 36$$
   $$4x^2 - y^2 = 4$$

50. Solve the following system of equations:

   $$\frac{1}{x} + \frac{3}{y} = 4$$

   $$\frac{2}{x} - \frac{1}{y} = 7$$