Test III Calculus I

Place your name in the upper right hand corner. Place your answers in the blank spaces on the left. Show your work on a separate sheet of paper. Staple your work sheet(s) to the test when you hand it in. You may use your calculator, the trig. identities, and the area via summation sheets. Please enjoy yourself, calculus is fun.

1) Use the summation technique to find the area under the curve $y = mx + b$ between $x = a$ and $x = 2a$.

2) The following questions are true/false (on a math test?) they are not difficult but, please be careful. ‘$n$’ is any natural number. Graph them if confused.

   a) $0 = \int_{0}^{\pi/4} \tan(\theta) \, d\theta$

   b) $0 = \int_{0}^{2\pi} \cos(\theta) \, d\theta$

   c) $0 = \int_{0}^{\pi/3} \sin(\theta) \, d\theta$

   d) $0 = \int_{0}^{x^{2n}} \left( x^{2n+2} \right) \, dx$

   e) $0 = \int_{0}^{x^{2n}} \left( x^{2n+2} \right) \, dx + 2 \int_{0}^{x^{2n+2}} \left( x^{2n} \right) \, dx$

   f) $0 = \int_{0}^{x^{4n+1}} \left( x^{4n+1} \right) \, dx$

   g) $0 = \int_{0}^{x^{3n+1}} \left( x^{3n+1} \right) \, dx + 2 \int_{0}^{x^{3n+1}} \left( x^{3n+1} \right) \, dx$

3) A rectangular beam is to be cut from a cylindrical log of radius ‘$r$’.

   a) Show that the beam of maximal cross-sectional area is a square.

   b) Suppose that the strength of a rectangular beam is proportional to the product of the width and the cube of the depth. Find the dimensions of the strongest beam that can be cut from the cylindrical log.

4) What is the area under the curve $y = x^{11} + x$, between $x = a$ and $x = 2a$? $a > 0$.

5) Compute the following derivatives.

   a) $y = \ln(e^{3x})$

   b) $y = \ln(x^2 \cdot 4x + 4)$

   c) $y = \ln\left( \frac{1}{x^2 \cdot 6x + 9} \right)$

   e) $y = e^{2\ln(x)}$

   f) $y = \frac{\ln(x)}{e^x}$

   d) $y = e^{x^2 \cdot 2x + 1}$
6) Compute the following integrals

a) \[ \int \frac{x^2}{x^2 - 2} \frac{2}{4x + 4} \, dx \]

b) \[ \int x^2 e^{x^3} \, dx \]

c) \[ \int \frac{dx}{x^n} \]

d) \[ \int 2x^3 \, dx \]

7) Draw a curve with the following properties: on \((a, b)\) \(y' > 0\) and \(y'' > 0\), on \((b, c)\) \(y' < 0\) and \(y'' < 0\), on \((c, d)\) \(y' < 0\) and \(y'' > 0\), on \((d, e)\) \(y' > 0\) and \(y'' > 0\), and on \((e, \infty)\) \(y' > 0\) and \(y'' < 0\). Please assume that \(a < b < c < d < e\).

8) If \(f(x)\) is a differentiable function such that \[ \int f(t) \, dt = \left( f(x) \right)^2 \] for all \(x\), find \(f\).

9) If \(f(x)\) is continuous and \(g(x)\) and \(h(x)\) are differentiable functions compute the following derivative.

\[ \frac{d}{dx} \left[ \int_{1}^{x} f(t) \, dt \right] \]