Test I Math 251

Place your name in the upper right hand corner. Place your answers on your work sheet in a clearly marked area. Staple the work sheet to your test when you hand it in. You may use your calculator, MAPLE, and the handouts I have given you. Relax, enjoy and have fun. 😊

1) A dietitian is preparing a meal consisting of foods A, B, and C. Each ounce of A contains 3 units of protein, 2 units of fat, and 4 units of carbohydrate. Each ounce of B contains 2 units of protein, 3 units of fat, and 1 unit of carbohydrate. Each ounce of C contains 4 units of protein, 3 units of fat, and 2 units of carbohydrate. If the meal must provide exactly 25 units of protein, 24 units of fat, and 21 units of carbohydrate, how many ounces of A, B, and C must be used?

2) Find the cubic polynomial that interpolates the points (1,3), (2,4), (-1, -3) and (3,7).

3) Find all the solutions to the given system of equations.

\[
\begin{align*}
x + y + 2z &= 1 \\
2x + 2y + z &= 2 \\
3x + 3y + 3z &= 3
\end{align*}
\]

4) Find all values of ‘a’ for which the system has, no solution, a unique solution, and infinitely many solutions.

\[
\begin{align*}
x + y &= 4 \\
x + (a^2 - 15)y &= a
\end{align*}
\]

5) Find the inverse for the following matrix.

\[
A = \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

6) Solve the following system for (x,y,z).

\[
\begin{align*}
x + y + 2z &= a \\
x + 2y + z &= b \\
2x + y + z &= c
\end{align*}
\]

7) Solve the following circuit for the currents.
8) Consider 4 boxes of jumping beans with the following transition probabilities for 1 minute intervals. Beans can only jump between adjacent boxes.

\[
\begin{align*}
1 \rightarrow 2 & \quad \text{is } \ 25\% \\
2 \rightarrow 3 & \quad \text{is } \ 30\% \\
2 \rightarrow 4 & \quad \text{is } \ 20\% \\
4 \rightarrow 2 & \quad \text{is } \ 50\% \\
3 \rightarrow 4 & \quad \text{is } \ 10\%
\end{align*}
\]

Write down the probability matrix. If we start out with 10 beans in each box what will their distribution be after 3 minutes? If we allow them to jump forever what will their final distribution be? Find a vector that describes all possible steady state solutions.

8) Consider the following set of matrices:

\[
\begin{align*}
\sigma_x &= \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \\
\sigma_y &= \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \\
\sigma_z &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\end{align*}
\]

Show the following:

\[
\begin{align*}
\sigma_x^2 &= \mathbb{I} \\
\sigma_y^2 &= \mathbb{I} \\
\sigma_y^2 &= \mathbb{I}
\end{align*}
\]

\[
\sigma_x \sigma_y \sigma_x \sigma_z = 2i \sigma_z \\
\sigma_x \sigma_z \sigma_x \sigma_y = 2i \sigma_y \\
\sigma_y \sigma_z \sigma_y \sigma_x = 2i \sigma_x
\]

\[
\det(\sigma_x) = 1 \\
\det(\sigma_y) = 1 \\
\det(\sigma_z) = 1
\]

In case you are wondering, this set of matrices has a special name. They are the Pauli spin matrices. They are used in Quantum Mechanics.